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Hydromagnetic Heat and Mass Transfer Flow with Heat Source and Time Dependent Suction in Rotating System

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ABSTRACT

In the present paper, heat and mass transfer effects on the unsteady flow of an incompressible, homogeneous, electrically conducting, viscous fluid through a time dependent porous medium past an infinite porous vertical plate with time dependent suction velocity under the influence of uniform magnetic field is studied. It is considered that the permeability of the porous medium decreases exponential with time. Using perturbation technique suggested by Lighthill (1954), the solutions for primary velocity, secondary velocity, temperature field and concentration field are obtained. In addition, expressions for skin-friction, rate of heat transfer, rate of mass transfer are also derived. The effects of important parameters on primary velocity and secondary velocity are observed with the help of figure. The effects of various parameters on skin-friction due to primary velocity and secondary velocity are discussed with the help of tables.

Keywords: MHD, Uniform Magnetic Field, Porous Vertical Plate, Unsteady Flow, Skin Friction

1. Introduction:-

In nature and in industries, many transport processes exists, where the transfer of heat and mass takes place as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. The phenomenon of heat and mass transfer also takes place in chemical processing industry such as food processing and polymer production. Due to application in engineering and technology, heat and mass transfer effects in flow of fluids have been investigated by so may authors.

A comprehensive study on the theory of rotating fluids has been presented by Greensapan (1968). Several authors including Shivprasad et. al. (1986), Hatzikonstantinou (1990), Jha (1991), Singh & Kulshreshta (1993), Singh (1994) Sattar and Slam (1995) Seth & Benerji (1996), Singh et al. (1999). Recently Singh et.

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al. (2001) have presented an analysis on free connection effects in the unsteady flow of an incompressible, electrically conducting, viscous, rotating liquid in a homogeneous porous medium, past an isothermal vertical porous plate with constant suction velocity normal to the plate under the influence of uniform magnetic field applied perpendicular to the flow. More recently, Singh et al. (2002) have studied unsteady oscillatory flow of an incompressible, electrically conducting and viscous liquid through a porous medium past an infinite vertical porous plate with constant suction and transverse uniform magnetic field. In this study the suction velocity and the permeability of the medium is assumed constant. Mishra (2003) was discussed a joint effects of magnetic field and electric field an generalized coutte flow with heat transfer.

In the present paper, heat and mass transfer effects on the unsteady flow of an incompressible, homogeneous, electrically conducting, viscous fluid through a porous medium with time dependent permeability past an infinite porous vertical plate with time dependent suction velocity under the influence of uniform magnetic field is studied. It is considered that the permeability of the porous medium decreases exponential with time. Using perturbation technique suggested by Lighthill (1954), the solutions for primary velocity, secondary velocity, temperature field and concentration field are obtained. In addition, expressions for skin-friction, rate of heat transfer, rate of mass transfer are also derived. The effects of important parameters on primary velocity and secondary velocity parameters on skin-friction due to primary velocity and secondary velocity are discussed with the help of tables.

2. Formulation of the Problem:-

Consider an unsteady oscillatory free and forced convective flow of an electrically conducting, homeogeneous, incompressible, viscous liquid through a porous medium past an infinite porous vertical plate in the presence of uniform magnetic field. Under Cartesian coordinate system (x, y, z), the plate is considered in z-plane. Initially, when $t \leq 0$ the plate and the fluid are assumed to be at the same temperature T_∞ and the foreign mass is assumed to be uniformly in the flow region such that it is every where C_∞. When, t>0, the temperature of the plate is instantaneously raised to T_w and the species concentration is also raised to C_w and thereafter maintained constant. In addition, the analysis, the analysis is based on the following assumptions:

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(i) The liquid and the plate both are in a state of rigid body rotation with uniform angular velocity Ω about z-axis.
(ii) The x-axis and y-axis are in the plane of two dimensional infinite vertical porous plates and z-axis is normal to these axes at the point of intersection.
(iii) The components of velocity in these directions are u,v, w respectively.
(iv) The suction velocity at the porous plate is time dependent i.e. $w=-w_0\left(1+\in e^{\text{int}}\right)$ where *n* is real and w_0 real and positive.
(v) The porous medium is time dependent i.e. $k(t)k_0\left(1+\in e^{\text{int}}\right)$ where *n* is real and positive.
(vi) A uniform magnetic field $\vec{B}_0=\mu_e\vec{H}$ so that [$\vec{H}=(0,0,H_0)$is the applied magnetic field vector] acts in the z-direction i.e. normal to the flow of fluid.
(vii) The magnetic Reynolds number is very small so that induced magnetic field is negligible in comparison to applied magnetic field.
(viii) No external electric field is applied in the flow region so that the effect of polarization of ionized fluid is negligible.
(ix) The Hall Effect and viscous dissipation effect have been ignored.
(x) Only electromagnetic body force (Lorentz force) is considered.
(xi) The heat source parameter is absorption type $Q=Q_0(T-T_\infty)$.
(xii) The usual Boussinesq's approximation is taken into account.
(xiii) The foreign mass is present at low level and uniformly distribution in the flow region.

Under the above stated assumptions, the equations of continuity is $\vec{\nabla}.\vec{q}=0$

Where $\vec{q}=(u,v,w)$ *and* $w=-w_0\left(1+\in e^{\text{int}}\right)$

Hence, the governing equations for the present configuration are:

$$\frac{\partial u}{\partial t}-w_0\left(1+\in e^{\text{int}}\right)\frac{\partial u}{\partial z}-2\Omega v=\upsilon=\frac{\partial^2 u}{\partial z^2}+g\beta^*(T-T_\infty)+\beta(C-C_\infty)$$

$$-\frac{\upsilon}{k_0\left(1+\in e^{\text{int}}\right)}u-\frac{\sigma}{\rho}\mu_e^2H_0^2u \quad \ldots..(1)$$

$$\frac{\partial v}{\partial t} - w_0\left(1+\in e^{\text{int}}\right)\frac{\partial v}{\partial z} - 2\Omega u = \upsilon\frac{\partial^2 v}{\partial z^2} - \frac{\upsilon}{k_0\left(1+\in e^{\text{int}}\right)}v - \frac{\sigma}{\rho}\mu_e^2 H_0^2 u \qquad \ldots..(2)$$

$$\frac{\partial T}{\partial t} - w_0\left(1+\in e^{\text{int}}\right)\frac{\partial T}{\partial z} = \frac{K}{\upsilon\rho C_p}\frac{\partial^2 T}{\partial z^2} - \frac{Q(T-T_\infty)}{\rho C_p}$$

$$\frac{\partial C}{\partial z} - w_0\left(1+\in e^{\text{int}}\right)\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} \qquad \ldots...(3)$$

where υ is the kinematic coefficient of viscosity, β^* is the volumetric coefficient of thermal expansion, β is the volumetric coefficient of thermal expansion with concentration, σ is the electrical conductivity of the liquid, ρ is the density of the liquid, ρ is the density of the liquid, μ_e is the magnetic permeability, H_0 in the uniform magnetic field k_0 is the constant permeability of the porous medium, K is the thermal conductivity, C_p is the specific heat at constant pressure, T is the temperature T_w is the plate temperature, C_w is the concentration at the plate, C_∞ is the concentration of species far away from the plate, T_∞ is the temperature far away from the plate, D is the thermal diffusivity and other symbols have their usual meaning.

The boundary conditions for the present problem are:

$$u=0, \quad v=0, \quad T=T_w\left(1+\in e^{\text{int}}\right), \quad C=C_w\left(1+\in e^{\text{int}}\right)\ at\ z=0 \qquad \ldots.\ (4)$$

$$u\to 0 \quad v\to 0, \quad T\to T_\infty, \quad C\to C_\infty \quad as \quad z\to\infty \qquad \ldots..(5)$$

We introduce the following non-dimensional quantities:

$$u^* = \frac{u}{U_0}, \quad v^* = \frac{v}{U_0}, \quad z^* = \frac{w_0 z}{\upsilon}, \quad t^* = \frac{w_0^2 t}{\upsilon}, \quad K^* = \frac{w_0^2 K}{\upsilon^2},$$

$$k_0{}^* = \frac{w_0^2 k_0}{\upsilon^2}, \quad W^* = \left(\frac{u}{U_0} + i\frac{v}{U_0}\right), \quad n^* = \frac{\upsilon n}{w_0^2} \quad and \quad T^* = \frac{T-T_\infty}{T_w - T_\infty}$$

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Using these non-dimensional quantities, the equation (1), (2), (3) and (4) reduce to :

$$\frac{\partial u}{\partial t}-\left(1+\in e^{\mathrm{int}}\right)\frac{\partial u}{\partial z}-2Ev=G_rT=G_mC\frac{\partial^2 u}{\partial z^2}-\left[M^2+\frac{1}{k_0\left(1+\in e^{\mathrm{int}}\right)}\right]u \quad .....(6)$$

$$\frac{\partial v}{\partial t}-\left(1+\in e^{\mathrm{int}}\right)\frac{\partial u}{\partial z}-2Eu=\frac{\partial^2 v}{\partial z^2}-\left[M^2+\frac{1}{k_0\left(1+\in e^{\mathrm{int}}\right)}\right]v \quad .....(7)$$

$$P_r\frac{\partial T}{\partial t}-P_r\left(1+\in e^{\mathrm{int}}\right)\frac{\partial T}{\partial z}=\frac{\partial^2 T}{\partial z^2}-\alpha_0 T \quad .....(8)$$

$$S_C\frac{\partial C}{\partial t}-S_C\left(1+\in e^{\mathrm{int}}\right)\frac{\partial C}{\partial z}=\frac{\partial^2 C}{\partial z^2} \quad .....(9)$$

where $P_r=\frac{\mu C_p}{K}$ (Prandtl number),

$S_c=\frac{\upsilon}{D}$ (Schmidt number)

$G_r=\frac{\upsilon g\beta^*\left(T_w-T_\infty\right)}{w_0^2U_0}$ (Grashof number),

$G_m=\frac{\upsilon g\beta^*\left(C_w-C_\infty\right)}{w_0^2U_0}$ (Modified Grashof number)

$E=\frac{\upsilon\Omega}{w_0^2}$ (Rotational parameter),

$M=\frac{\mu_e H_0}{w_0}\sqrt{\frac{\sigma\upsilon}{\rho}}$ (Hartmann number) and $\alpha_0 = \dfrac{\upsilon^2 Q_0}{K w_0^2}$ (Heat source parameter)

Combining (6) and (7) and using u + iv = W, we obtain

$$\frac{\partial W}{\partial t} - \left(1 + \in e^{\text{int}}\right)\frac{\partial W}{\partial z} + 2iEW + \left(M^2 + \frac{1}{k_0\left(1 + \in e^{\text{int}}\right)}\right)W$$

$$= \frac{\partial^2 W}{\partial z^2} + G_r T + G_m C \qquad \text{.....(10)}$$

The boundary condition (5) reduced to

$$W = 0 \qquad T = 1 + \in L_1 e^{\text{int}} \qquad C = 1 + \in L_2 e^{\text{int}} \qquad at \quad z = 0$$

$$W \to 0, \quad T \to 0, \qquad C \to 0, \qquad as \qquad z \to \infty \qquad \text{.....(11)}$$

$$L_1 = \frac{T_w}{T_w - T_\infty} \qquad and \qquad L_2 = \frac{C_w}{C_w - C_\infty}$$

3. Solution of the Problem:-

Following Lighthill (1954), we assume the velocity and temperature of the liquid in the neighbourhood of the plate as:

$$W(z,t) = W_1(z) + \in W_2(z) e^{\text{int}} \qquad \text{.........(12)}$$

$$T(z,t) = T_1(z) + \in T_2(z) e^{\text{int}} \qquad \text{........(13)}$$

$$C(z,t) = C_1(z) + \in C_2(z) e^{\text{int}} \qquad \text{.........(14)}$$

Using (12), (13) and (14) in (8), (9) and (10), we obtain:

$$W_1''(z) + W_1'(z) - (M_1 + 2iE)W_1(z) = -G_r T_1(z) - G_m C_1(z) \qquad \text{..... (15)}$$

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$$W_2''(z)+W_2'(z)-[M_1+(2E+n)]W_2(z)=-G_rT_2(z)-G_mC_2(z)-W_1'(z)\frac{1}{k_0}W_1 \quad \text{..... (16}$$

$$T_1''(z)+P_rT_1'(z)-\alpha_0T_1(z)=0 \quad \text{..... (17)}$$

$$T_2''(z)+P_rT_2'(z)-(inP_r+\alpha_0)T_2(z)=-P_rT_1'(z) \quad \text{..... (18)}$$

$$C_1''(z)+S_CC_1'(z)=0 \quad \text{..... (19)}$$

$$C_2''(z)+S_CC_2'(z)inS_CC_2=-S_CC_1'(z) \quad \text{..... (20)}$$

where $M_1=M^2+\frac{1}{k_0}$

Using (12), (13) and (14) in (11) the boundary conditions become:

$$W_1=0,\ W_2=0,\quad T_1=1,\quad T_2=L_1,\quad C_1=1,\quad C_2=L_2\quad at\ z=0$$

$$W_1=0,\ W_2=0,\ T_1=0,\quad T_2=0,\quad C_1=0,\quad C_2=0,\quad as\ z=\infty \quad \text{..... (21)}$$

The solution of the coupled equations (15) – (20) under the boundary conditions (21) are :

$$T_1(z)=e^{-M_2z} \quad \text{..... (22)}$$

$$T_2(z)=(L_1-F_1)e^{-M_4z}+F_1e^{-M_2z} \quad \text{..... (23)}$$

$$C_1(z)=e^{-S_Cz} \quad \text{..... (24)}$$

$$C_2(z)=L_2e^{-R_3z}+\frac{S_c}{L_2}\left(e^{-R_3z}-e^{-S_cz}\right) \quad \text{..... (25)}$$

$$W_1(z)=F\left(e^{-M_2z}-e^{-M_6z}\right) \quad \text{..... (26)}$$

and $W_2(z)=F_3e^{-M_2z}+F_4e^{-M_4z}+F_5e^{-M_6z}$

$$+ R_5 e^{-R_3 z} + R_6 e^{M_6 z} + R_7 e^{-S_c z}$$

$$+ \left(F_3 + F_4 + F_5 + R_6 + R_6 + R_7\right) e^{-M_8 z} \qquad \ldots.. (27)$$

where $R_3 = \dfrac{S_c + \sqrt{S_c^2 + 4inS_c}}{2}$, $\qquad M_1 = M^2 + \dfrac{1}{k_0}$

$$M_2 = \frac{P_r + \sqrt{P_r^2 + 4\alpha_0}}{2},$$

$$M_4 = A_1 + iB_1 = \frac{P_r + \sqrt{P_r^2 + 4(inP_r + \alpha_0)}}{2},$$

$$M_6 = A_2 + iB_2 = \frac{1 + \sqrt{1 + 4M_1 + 8iE}}{2},$$

$$M_8 = A_3 + iB_3 = \frac{1 + \sqrt{1 + 4M_1 + 4i(2E + n)}}{2},$$

$$F_1 = (A_4 + iB_4) = \frac{M_2 P_r}{(M_2^2 - M_2 P_r - \alpha_0) - i(nP_r)},$$

$$F_2 = (A_5 + iB_5) = \frac{-G_r}{(M_2^2 - M_2 - M_1) - i(2E)},$$

$$F_3 = (A_6 + iB_6) = \frac{(k_0 M_2 - 1)F_2 - k_0 G_r F_1}{k_0\left[(M_2^2 - M_2 - M_1) - i(2E + n)\right]},$$

$$F_4 = (A_7 + iB_7) = \frac{G_r (F_1 - L_1)_1}{\left[(M_2^2 - M_4 - M_1) - i(2E + n)\right]},$$

$$F_5 = (A_7 + iB_7) = \frac{G_r (F_1 - L_1)_1}{\left[(M_2^2 - M_4 - M_1) - i(2E + n)\right]},$$

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$$R_3 = P_1 + iQ_2 = \frac{S_c + \sqrt{S_c^2 + 4inS_c}}{2},$$

$$R_4 = P_2 + iQ_2 = \frac{-G_m}{S_c^2 - S_c - M_1 - i2E},$$

$$R_5 = P_3 + iQ_3 = \frac{-nG_m L_2 + iG_m S_c}{n\left[R_c^2 - R_3 M_1 - i(2E+n)\right]},$$

$$R_6 = P_4 + iQ_4 = \frac{(M_6 k_0 + 1)R_4}{k_0\left[M_6^2 - M_6 - M_1 - i(2E+n)\right]},$$

$$R_7 = P_5 + iQ_5 = \frac{-nR_4 k_0 S_c - nR_4 - iG_m k_0 S_c}{nk_0\left[S_c^2 - S_c - M_1 - i(2E+n)\right]},$$

$$A_1 = \frac{P_r}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{\left(P_r^2 + 4\alpha_0\right)^2 + 16n^2 P_r^2} + \left(P_r^2 + 4\alpha_0\right)\right]^{1/2},$$

$$B_1 = \frac{1}{2\sqrt{2}}\left[\sqrt{\left(P_r^2 + 4\alpha_0\right)^2 + 16n^2 P_r^2} - \left(P_r^2 + 4\alpha_0\right)\right]^{1/2},$$

$$A_2 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M_1)^2 + 64E^2} + (1+4M_1)\right]^{1/2},$$

$$B_2 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M_1)^2 + 64E^2} - (1+4M_1)\right]^{1/2},$$

$$A_3 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M_1)^2 + 16(2E+n)^2} + (1+4M_1)\right]^{1/2},$$

$$B_3 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M_1)^2 + 16(2E+n)^2} - (1+4M_1)\right]^{1/2},$$

$$P_1 = \frac{S_c}{2} + \frac{1}{2\sqrt{2}}\left[S_c\sqrt{S_c^2 + 16n^2} + S_c^2\right]^{1/2},$$

$$Q_1 = \frac{1}{2\sqrt{2}}\left[S_c\sqrt{S_c^2 + 16n^2} - S_c^2\right]^{1/2},$$

$$P_2 = \frac{-G_m G_1}{G_1^2 + 4E^2}, \qquad Q_2 = \frac{-2G_m E}{G_1^2 + 4E^2},$$

$$P_3 = \frac{G_4}{n\left(G_1^2 + G_3^2\right)}, \qquad Q_3 = \frac{G_5}{n\left(G_2^2 + G_3^2\right)},$$

$$P_4 = \frac{G_{10}}{G_6^2 + G_7^2}, \qquad Q_4 = \frac{G_{11}}{G_6^2 + G_7^2},$$

$$P_5 = \frac{G_{14}}{nk_0\left[G_1^2 + (2E+n)^2\right]}, \qquad Q_5 = \frac{-G_{15}}{nk_0\left[G_1^2 + (2E+n)^2\right]},$$

$$G_1 = S_c^2 - S_c - M_1, \qquad G_2 = P_1^2 - Q_1^2 - P_1 - M_1,$$

$$G_3 = 2P_1Q_1 - Q_1 - 2E - n, \qquad G_4 = -G_m G_2 n L_2 + G_m G_3 S_c,$$

$$G_5 = -G_m G_2 S_c + G_m G_3 n l_2, \qquad G_6 = k_0\left(A_1^2 - B_1^2 - A_2 - M_1\right),$$

$$G_7 = k_0(2A_2B_2 - B_2 - 2E - n) \qquad G_8 = (A_2k_0 + 1)P_2 - B_2k_0Q_2,$$

$$G_9 = (A_2k_0 + 1)Q_2 - B_2k_0Q_2, \qquad G_{10} = G_6G_8 + G_7G_9,$$

$$G_{11} = G_6G_9 + G_7G_8, \qquad G_{12} = -nP_2(k_0S_c + 1),$$

$$G_{13} = nQ_2(k_0S_c + 1) + G_m k_0 S_c, \qquad G_{14} = G_{12}G_1 + G_{13}(2E+n),$$

$$G_{15} = G_{13}G_1 - G_{12}(2E+n),$$

$$A_4 = \frac{M_2 P_r a_0}{a_0^2 + n^2 P_r^2}, \qquad B_4 = \frac{n M_2 P_r}{a_0^2 + n^2 P_r^2},$$

$$A_5 = \frac{-G_r b_0}{b_0^2 + 4E^2}, \qquad B_5 = \frac{-2G_r E}{b_0^2 + 4E^2},$$

$$A_6 = \frac{d_3}{k_0\left[b_0^2 + (2E+n)^2\right]}, \qquad B_6 = \frac{d_4}{k_0\left[b_0^2 + (2E+n)^2\right]},$$

$$A_7 = \frac{a_2}{a_1^2 + b_1^2}, \qquad B_7 = \frac{b_2}{a_1^2 + b_1^2},$$

$$A_8 = \frac{a_3a_4 + b_3b_4}{a_4^2 + b_4^2}, \qquad B_8 = \frac{b_3a_4 - a_3b_4}{a_4^2 + b_4^2},$$

$$A_9 = A_6 + A_7 + A_8 + P_3 + P_4 + P_5,$$

$$B_9 = B_6 + B_7 + B_8 + Q_3 + Q_4 + Q_5,$$

$$a_0 = M_2^2 - M_2 P_r - \alpha_0, \qquad b_0 = M_2^2 - M_2 P_r - M_1,$$

$$a_1 = A_1^2 - B_1^2 - A_1 - M_1, \qquad b_1 = 2A_1B_1 - B_1 - 2E - n,$$

$$a_2 = G_r a_1 (A_4 - L_1) + G_r B_4 b_1, \qquad b_2 = G_r B_4 a_1 - G_r b_1 (A_4 - L_1),$$

$$a_3 = A_5 - k_0 (A_2 A_5 - B_2 B_5), \qquad b_3 = B_5 - k_0 (B_2 B_5 - A_2 A_5),$$

$$a_4 = k_0 \left(A_2^2 - B_2^2 - A_2 - M_1\right), \qquad b_4 = k_0 (2A_2B_2 - B_2 - 2E - n),$$

$$d_1 = k_0 (M_2 A_5 - G_r A_4) - A_5, \qquad d_2 = k_0 (M_2 B_5 - G_r B_4) - B_5,$$

$$d_3 = b_0 d_1 - d_2 (2E + n), \qquad d_4 = b_0 d_2 + d_1 (2E + n),$$

Substitution $T_1(z), T_2(z), C_1(z), C_2(z), W_1(z),$ and $W_2(z)$ in (12) – (14), we obtain:

$$T(z,t) = e^{-M_2 z} + \in \left[(L_1 - F_1) e^{-M_4 z} + F_1 e^{-M_2 z} \right] e^{\mathrm{int}} \qquad \text{.....(28)}$$

$$C(z,t) = e^{-S_c z} + \in \left[L_2 e^{-R_3 z} + \frac{S_c}{in} \left(e^{-R_3 z} - e^{-S_c z} \right) \right] e^{\mathrm{int}} \qquad \text{.....(29)}$$

$$W(z,t) = F_2 \left(e^{-M_2 z} - e^{-M_6 z} \right) + R_4 \left(e^{-S_c z} - e^{-M_6 z} \right)$$
$$+ \in \left[F_3 e^{-M_2 z} + F_4 e^{-M_4 z} + F_5 e^{-M_6 z} \right.$$
$$+ R_5 e^{-R_3 z} + R_6 e^{-M_6 z} + R_7 e^{-S_c z}$$
$$\left. - (F_3 + F_4 + F_5 + R_5 + R_6 + R_7) e^{-M_8 z} \right] e^{\mathrm{int}} \qquad \text{.....(30)}$$

From (30), the steady part of the primary velocity $u_1(z)$ and the steady part of the secondary velocity $v_1(z)$ are :

$$u_1(z) = A_5 e^{-M_2 z} - (A_5 \cos B_2 z + B_5 \sin B_2 z) e^{-A_2 z} + P_2 e^{-S_c z} - (P_2 \cos B_2 z + Q_2 \sin B_2 z) e^{-A_2 z} \quad .....(31)$$

$$v_1(z) = B_5 e^{-M_2 z} - (B_5 \cos B_2 z + A_5 \sin B_2 z) e^{-A_2 z} + Q_2 e^{-S_c z} - (Q_2 \cos B_2 z + P_2 \sin B_2 z) e^{-A_2 z} \quad .....(32)$$

From (30), the unsteady part i.e. time dependent part of the primary velocity $u_2(z)$ and the unsteady that i.e. time dependent part of the secondary velocity $v_2(z)$ are :

$$u_2(z) = (A_7 \cos B_1 z + B_7 \sin B_1 z) e^{-A_1 z} - (A_8 \cos B_2 z + B_8 \sin B_2 z) e^{-A_2 z} - (A_9 \cos B_3 z + B_9 \sin B_3 z) e^{-A_3 z} + A_6 e^{-M_2 z} + (P_4 \cos B_2 z + Q_4 \sin B_2 z) e^{-A_2 z} + P_5 e^{-S_c z} - (P_3 \cos Q_1 z + Q_3 \sin Q_1 z) e^{-P_1 z} \quad .....(33)$$

$$v_2(z) = (B_7 \cos B_1 z - A_7 \sin B_1 z) e^{-A_1 z} + (B_8 \cos B_2 z - A_8 \sin B_2 z) e^{-A_2 z} - (B_9 \cos B_3 z - A_9 \sin B_3 z) e^{-A_3 z} + B_6 e^{-M_2 z} + (Q_4 \cos B_2 z + P_4 \sin B_2 z) e^{-A_2 z} + Q_5 e^{-S_c z} - (Q_3 \cos Q_1 z + P_3 \sin Q_1 z) e^{-P_1 z} \quad .....(34)$$

Therefore, for these values of $u_1(z)$, $v_1(z)$, $u_2(z)$ and $v_2(z)$, the primary velocity $u(z,t)$ and secondary velocity $v(z,t)$ can be written as :

$$u(z,t) = u_1(z) + \in (u_2 \cos nt - v_2 \sin nt) \quad .....(35)$$

$$v(z,t) = v_1(z) + \in (v_2 \cos nt - u_2 \sin nt) \quad .....(36)$$

4. Skin-Friction, Rate of Heat Transfer and Mass Transfer:-

The non-dimensional skin-friction (τ) at the plate at z = 0 is:

$$\tau = \left(\frac{\partial W}{\partial z}\right)_{z=0} = \left(\frac{\partial W_1}{\partial z}\right)_{z=0} + \in e^{\mathrm{int}}\left(\frac{\partial W_2}{\partial z}\right)_{z=0} = \tau_p + i\tau_s \qquad .....(37)$$

Hence, primary skin-friction (τ_p) due to primary velocity is:

$$\tau_p = A_{10} + \in \left(A_{11}\cos nt - B_{11}\sin nt\right) \qquad .....(38)$$

Hence, secondary skin-friction (τ_s) due to secondary velocity is:

$$\tau_s = B_{10} + \in \left(B_{11}\cos nt - A_{11}\sin nt\right) \qquad .....(39)$$

where

$$A_{10} = A_5\left(A_2 - M_2\right) - B_2 B_5 + P_2\left(A_2 - S_c\right) - Q_2 B_2,$$

$$B_{10} = B_5\left(A_2 - M_2\right) + A_3 B_2 + Q_2\left(A_2 - S_c\right) + P_2 B_2,$$

$$A_{11} = A_3 A_9 - B_3 B_9 - A_1 A_7 + B_1 B_7 - A_2 A_8 + B_2 B_8 - M_2 A_6,$$
$$- P_1 P_3 + Q_1 Q_3 - A_2 P_4 - B_2 Q_4 - S_c P_5$$

And $$B_{11} = B_3 A_9 - A_3 B_9 - B_1 A_7 + A_1 B_7 - B_2 A_8 + A_2 B_8 - M_2 B_6,$$
$$- Q_1 P_3 + P_1 Q_3 - B_2 P_4 - A_2 Q_4 - S_c Q_5$$

Also, the rate of heat transfer at the plate at z=0 in terms of Nusselt number (N_u) is:

$$N_u = \left(\frac{\partial T}{\partial z}\right)_{z=0} = \left(\frac{\partial T_1}{\partial z}\right)_{z=0} + \in e^{\mathrm{int}}\left(\frac{\partial T_2}{\partial z}\right)_{z=0} \qquad .....(40)$$

Hence, considering that the real part only is of significance, we get:

$$N_u = -M_2 - \in \left[A_{12}\cos nt - B_{12}\sin nt\right] \qquad .....(41)$$

where

$$A_{12} = A_1(L_1 - A_4) + B_1 B_4 + M_2 A_4,$$

$$B_{12} = B_1(L_1 - A_4) + A_1 B_4 + M_2 B_4,$$

Also, the rate of mass transfer at the plate at z = 0 in terms of Sherwood number (S_n) is:

$$S_h = \left(\frac{\partial C}{\partial z}\right)_{z=0} = \left(\frac{\partial C_1}{\partial z}\right)_{z=0} + \in e^{\text{int}}\left(\frac{\partial C_2}{\partial z}\right)_{z=0} \qquad \text{.....(42)}$$

Hence, considering real part only, we get:

$$S_h = -S_c - \in \left[A_{13}\cos nt - B_{12}\sin nt\right] \qquad \text{.....(43)}$$

Where $A_{13} = P_1 L_2 + \frac{S_c Q_1}{n}$

$$B_{13} = Q_1 L_2 - \frac{P_1 S_c}{n} + \frac{S_c^2}{n}$$

Table – 1

Skin friction (τ_p) due to primary velocity

(t = 10, L_1 = 1.0, L_2 = 1.0 and ∈ = 0.002)

P_r	S_C	M	k_0	*n*	α_0	G_r	G_m	E	ι_p
0.71	0.22	1.0	10.0	5.0	1.0	6.00	12.0	1.0	9.97997
7.00	0.22	1.0	10.0	5.0	1.0	6.00	12.0	1.0	8.35971
0.71	0.60	1.0	10.0	5.0	1.0	6.00	12.0	1.0	8.92422
0.71	0.22	2.0	10.0	5.0	1.0	6.00	12.0	1.0	8.00216

0.71	0.22	1.0	40.0	5.0	1.0	6.00	12.0	1.0	9.97137
0.71	0.22	1.0	10.0	10.0	1.0	6.00	12.0	1.0	9.98468
0.71	0.22	1.0	10.0	5.0	2.0	6.00	12.0	1.0	9.68307
0.71	0.22	1.0	10.0	5.0	1.0	12.0	12.0	1.0	12.3661
0.71	0.22	1.0	10.0	5.0	1.0	6.00	18.0	1.0	13.7757
0.71	0.22	1.0	10.0	5.0	1.0	6.00	12.0	2.0	6.86091

Table – 5

Skin friction (τ_s) due to primary velocity

($t = 10$, $L_1 = 1.0$, $L_2 = 1.0$ and $\in = 0.002$)

P_r	S_C	M	k_0	n	α_0	G_r	G_m	E	τ_p
0.71	0.22	1.0	10.0	5.0	1.0	6.00	12.0	1.0	-5.84201
7.00	0.22	1.0	10.0	5.0	1.0	6.00	12.0	1.0	-5.13736
0.71	0.60	1.0	10.0	5.0	1.0	6.00	12.0	1.0	-4.00917
0.71	0.22	2.0	10.0	5.0	1.0	6.00	12.0	1.0	-1.83152
0.71	0.22	1.0	40.0	5.0	1.0	6.00	12.0	1.0	-6.03315
0.71	0.22	1.0	10.0	10.0	1.0	6.00	12.0	1.0	-5.83082
0.71	0.22	1.0	10.0	5.0	2.0	6.00	12.0	1.0	-5.64763
0.71	0.22	1.0	10.0	5.0	1.0	12.0	12.0	1.0	-6.58272
0.71	0.22	1.0	10.0	5.0	1.0	6.00	18.0	1.0	-8.39266
0.71	0.22	1.0	10.0	5.0	1.0	6.00	12.0	2.0	-5.16182

5. Discussion and Conclusions:-

The effects of Schmidt number (S_c), magnetic parameter (M), Grashof number (G_r), modified Grashof number (G_m) and rotation parameter (E) on primary velocity (u) and secondary velocity (v)at P_r =0.71m, k_o = 10.0, n=10.0, α_o =1.0, L_1 +1.0, L_2 =1.0 , t = 1.0 and ε = 0.002 are numerically observed and are shown in Fig-1 and Fig- 2. The effects of Prandtl number (P_r), Schmidt number (S_c), magnetic parameter (M), permeability parameter (k_o), frequency parameter (n), heat source parameter (α_o), Grashof number (G_r), modified Grashof number (G_m) androtation parameter (E) on skin-friction (τ_p) due to primary velocity and skin-friction (τ_s) due to secondary velocity at the plate at L_1 = 1.0, L_2 =1.0, t = 1.0 and ε = 0.002 are observed numerically and are represented in Table-1 and Table-2 respectively. The conclusions of the study are as follows:

1. An increase in G_r or Gm increases primary velocity while an increase in Sc, M or E decreases the primary velocity.
2. An increase in G_r and M both, the primary velocity decreases.
3. An increase in G_m and E both, the primary velocity decreases.
4. The primary velocity increases near the plate and after attaining a maximum value it decreases as z increases.
5. An increase is S_c, M or E increase secondary velocity while an increase in G_r or G_m decreases the secondary velocity.
6. An increase in G_r and M both, the secondary velocity increases.
7. An increase in G_m and E both, the secondary velocity increase.
8. The secondary velocity decreases near the plate and after attaining a minimum value it increase as z increases.
9. An increase in P_r, S_c, M, n, α_0 *or* E decreases the skin-friction due to primary velocity while an increase in k_0, G_r or G_m increases the skin-friction due to primary velocity,

Increases in k_0, G_r or G_m increase the skin-friction due to secondary velocity while an increase in P_r, S_c, M, n, α_0 *or* E decreases the skin friction due to secondary velocity.

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